

Has toughness of local competition declined?

by

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Abstract

Recent evidence on firm-level markups and concentration raises a concern that market competition has declined in the U.S. over the last few decades. Since measuring competition is difficult, methodologies used to arrive at these findings have merits but also raise technical concerns which question the validity of these results. Given the significance of documenting how competition has changed, I contribute to this literature by studying a different measure of competition. Specifically, I estimate the toughness of local competition over time. To derive this estimate, I use a generalized monopolistic competition model with variable markups. This model generates insights that allows me to measure competition as the sensitivity of weighted average markup to changes in the number of competitors using directly observable variables. Compared to firm-level markups estimation, this method relaxes the need to estimate production functions. I then use confidential Census data to estimate toughness of local competition from 1997 to 2016, which shows that local competition has decreased in non tradable industries on average in the U.S. during this time period.

Keyword: Market size, local competition, markups

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1. Introduction

Competition is an engine of economic growth; it motivates firms to innovate to increase productivity, which leads to higher aggregate TFP. It is concerning that recent evidence on sale concentration, corporate profit and aggregate markup suggests that competition might have weakened in the U.S. However, such findings are often subject to controversy. For example, concentration is a measure of a lack of competition in the Cournot model. However, outside of this context, higher concentration doesn't necessarily indicate a less competitive market. On the other hand, markups are more often used to measure market power because they are closely related to the elasticities of demand. However, computing markups using the so-called production approach involves estimating the production functions, which imposes stringent data requirements.

Therefore, in this study I look at a different measure, the toughness of local competition. It indicates how the average markup declines when the number of competitors increases. Using a very general monopolistic competition model, I show how to estimate this measure with minimal data requirements and without estimating the production functions. I then show how my measure relates to the typical markup estimation method. Taking this method to the data, I find that toughness of local competition has decreased between 1997 and 2016.

I distinguish between local and national competition as in Rossi-Hansberg, Sarte, and Trachter (2020); in this study, local competition is the object of interest because most sales take place locally. A casual observation suggests that a supermarket in South Bend, Indiana is more likely to compete with similar stores nearby than those in Chicago. In addition, according to the Census 4th Quarter 2019 E-Commerce Sale report, 90 percent of retail sales are in store. In the academic literature, Gervais and Jensen (2019) use a gravity model and estimate that a large number of industries in the service sector have most of their outputs consumed locally (where outputs are produced). Taken together, these findings suggest that most economic activities are still local. Thus, this study focuses on local competition by providing empirical results solely for non-tradable industries. However, I show in the appendix that my theoretical result can be applied to tradable goods as well.

In order for my theory to be broadly applicable, I use a highly general monopolistic competition model based on Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018).

However, my theoretical result is not based on a specific productivity distribution such as the Pareto distribution but holds in a more general setting. In my model, the economy is made up of many isolated markets in which firms enter and sell locally. Firms charge variable markups depending on their elasticities of demand. The distribution of firms in a given market endogenously generates a cutoff productivity level that forces some firms to exit immediately. From the model, I define toughness of competition as an elasticity measuring the decline in weighted-average markup due to an increase in number of competitors, as in Sutton (1991). The model makes it conceptually simple to estimate this measure because it is simply equal to the ratio of the inverse-wage share (average revenue over average payroll) elasticity to market size and the number of firms elasticity to market size.

To measure toughness of local competition empirically, one can use the publicly available version of the Economic Census; however, for a broader coverage, I rely on the confidential Census data from 1997 to 2016. I measure a market size as a geographical area’s population, and I use the historical spatial distribution of population as an instrument for market size to account for some potential endogeneity concerns with population. In the baseline estimation, I define each market as a pair of county-6 digit 2012 NAICS. Overall, I find a decline in local competition over time; this finding is prevalent to most industries in the study and pass a series of robustness checks.

Related Literature: To my knowledge, this is the first study to estimate toughness of local competition over time. However, there is closely related literature as well. Some recent papers have estimated markups for publicly listed firms or manufacturing firms to infer about market competition. It is notoriously hard to estimate markups, even with a sound econometric method; one needs to know the cost structure of a firm to estimate markups and separate data on quantity and price to estimate the production functions. De Loecker, Eeckhout, and Unger (2020) use the production method to estimate firm-level markups with Compustat data and find an increase in revenue-weighted average markup since 1980. In practice, they rely on revenue elasticities instead of output elasticities, which leads to certain technical issues. For example, Bond, Hashemi, Kaplan, and Zoch (2021) show that in this method, markups are only identified when true markups are equal to 1 because revenue elasticities and output elasticities should be equal when firms do not have

influence on prices. Consequently, my research avoids this issue because I do not need to estimate the production functions.

Alternatively, the macroeconomics literature has looked at concentration as a measure of competition both nationally and locally. The benefit of using concentration is simplicity as one only needs to have data on firms' revenue. Autor, Dorn, Katz, Patterson, and Van Reenen (2020) estimate top four concentration for major sectors in the U.S. at the national level and find increases in sale and employment concentration since the early 1980s. They also argue that these increases in concentrations are consistent with the decline in labor share. However, they view this as evidence for a "winner takes all" environment, which reflects tougher competition. Grulon, Larkin, and Michaely (2019) document the increases in concentration and profit margins in American industries without an increase in operational efficiencies and attribute this to a weakened competition environment. Barkai and Benzell (2018) also find rising corporate profit rates since the 1980s. Rossi-Hansberg, Sarte, and Trachter (2020) estimate concentrations at the local and national levels using the National Establishment Time-Series dataset and find diverging trends, which they attribute to the expansion of large firms into new geographical markets. Smith and Ocampo (2021) also estimates these time series but finds opposite results for the retail sector using the Census data.

My paper is also related to a literature that studies competition across different market size. Asplund and Nocke (2006), Campbell and Hopenhayn (2005) and Syverson (2004) among others, provide empirical evidence on competition intensity across markets within a country. Melitz and Ottaviano (2008) propose a theoretical model about the selection force across international markets. Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018), which this work extends theoretically, is also a related paper in this strand.

The remainder of this paper is organized as follows. Section 2 presents the structural model to measure competition. Section 3 describes the data, and section 4 describes the empirical estimation step. Section 5 presents the main set of empirical results and the robustness checks, and section 6 concludes.

2. Theoretical model

This theoretical model is based on Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018) with an extension on the production side. The economy consists of many isolated regions that are different in the number of consumers/workers L but otherwise ex-ante identical; labor is exogenously supplied. In each region, firms sell differentiated non-tradable products (I show in Appendix B that the same theoretical result holds in an economy with tradable goods) and compete in a free-entry equilibrium.

A **consumer** with income i in a given location faces the vector of prices $\mathbf{p} = \{p_{\omega \in \Omega}\}$ where Ω is the set of varieties that are available, to be determined in equilibrium. The demand function for variety ω per consumer is given by:

$$q_{\omega}(\mathbf{p}, i) = Q D \left(\frac{p_{\omega}}{P(\mathbf{p}, i)} \right) \quad (1)$$

where Q and $P(\mathbf{p}, i)$ are demand shifters, $D(\cdot)$ is a strictly decreasing function.

Assumption: $\exists b$ such that $D(x) = 0$ for $x \geq b$

This assumption guarantees the existence of a “choke price”, above which the quantity demanded for a variety ω is 0. Normalize $b = 1$, $D(p_{\omega}/P(\mathbf{p}, i)) = 0$ if $p_{\omega} \geq P(\mathbf{p}, i)$. So the aggregate demand shifter $P(\mathbf{p}, i)$ will be equal to the choke price. This assumption limits the scope of utility functions to those that generate downward sloping demand curves with finite y-intercepts. An example of utility function that satisfies this assumption is Kimball (1995) as in Edmond, Midrigan, and Xu (2020). This assumption helps to generate endogenous exit without fixed costs, which is an analytical convenience.

Firms are monopolistically competitive; they produce differentiated varieties using Cobb-Douglas production functions with labor and capital according to

$$y = \frac{1}{c} l^{1-\alpha} k^{\alpha} \quad (2)$$

An entrant pays sunk entry cost f_E to draw the cost c from some distribution $G(c)$. Upon drawing, a firm enters if its expected profit is greater than or equal to the sunk entry cost, otherwise it exits immediately. A firm chooses its output level to maximize profit (indexing a firm by c instead of a variety since different varieties at the same price have the same effects on utility).

Firms maximize profit subject to the demand function (1), which yields the standard pricing function:

$$p = \frac{\epsilon(p/P)}{\epsilon(p/P) - 1} mc$$

where $\epsilon(p/P) = -\frac{\partial \ln(q(L, Q, P, p))}{\partial \ln(p)}$ is the elasticity of demand and mc is the marginal cost. The result that ϵ is a function of relative price can be seen by taking log of the demand function and differentiate wrt to p . Following Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018), I use the following change of variable, I denote the markup function $m = \frac{p}{mc}$ and define $\nu = \frac{P}{mc}$ as the relative (to the choke price) efficiency of a firm in the market. Using these new notations, elasticity of demand can be rewritten as $\epsilon(p/P) = \epsilon(m/\nu)$, and hence markup is pinned down by:

$$m = \frac{\epsilon(m/\nu)}{\epsilon(m/\nu) - 1}$$

To guarantee the uniqueness of markups, assuming $\epsilon' > 0$ (this is a well-known condition for markup to increase in productivity), then ϵ is strictly increasing and invertible, so for each ν , there is a unique m . Denoting $\mu(\nu) = m$, the following two properties of $\mu(\nu)$ hold: **(i)** a result of $\epsilon' > 0$ is that $\mu' > 0$, **(ii)** $P = mc^*$ where mc^* is the zero profit cutoff marginal cost level. The first property states that firms that face lower elasticities of demand charge higher markups; the second property states that a firm that charges at the choke price incurs no markup.¹

Toughness of local competition is defined as the decline in the equilibrium average markup due to an increase in number of firms as in Campbell and Hopenhayn (2005) or Sutton (1991). The model suggests that this measure can be derived from the elasticity of the inverse-wage share to market size and the elasticity of the number of firms to market share. The advantage of this measure is that while markups are not directly observed, the inverse-wage share and the number of firms are directly observable. To derive the measure, I proceed by writing out the analytical solutions of the two aforementioned elasticities.

Revenue function is given by:

$$r = \mu(\nu)(mc)LQD(\mu(\nu)/\nu).$$

¹See proof in Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2018), page 54.

Note that the residual demand function is multiplied with market size L . Ex-post average revenue is therefore:

$$\bar{r} = L Q \frac{P^2}{\left(\frac{w_k}{\alpha}\right)^\alpha \left(\frac{w_l}{1-\alpha}\right)^{(1-\alpha)}} \int_1^\infty \mu(\nu) D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^*(\nu) d\nu \quad (3)$$

The relevant object of interest is the cost-weighted average markup $\bar{\mu}$ as in Edmond, Midrigan, and Xu (2020). Weighted-average markup as opposed to un-weighted markup is appropriate because the recent literature finds that markups have been increasing disproportionately for large firms while the median markup has stayed flat (see e.g., De Loecker, Eeckhout, and Unger (2020)). Denoting \bar{TC} as the average cost used for production, and define the weighted average markup as:

$$\bar{\mu} = \int_0^{c^*} \frac{TC}{\bar{TC}} \mu \left(\frac{P}{mc} \right) g^*(c) dc,$$

which gives

$$\bar{\mu} = \frac{\int_1^\infty D(\mu(\nu)/\nu) \mu(\nu) \frac{1}{\nu^3} g^*(\nu) d\nu}{\int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^*(\nu) d\nu} \quad (4)$$

And since

$$\bar{l} = \left(\frac{1-\alpha}{\alpha} \frac{w_k}{w_l} \right)^\alpha L Q \frac{P^2}{\left(\left(\frac{w_k}{\alpha} \right)^\alpha \left(\frac{w_l}{1-\alpha} \right)^{(1-\alpha)} \right)^2} \int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^*(\nu) d\nu \quad (5)$$

Combining equations (3), (4) and (5) gives:

$$\bar{\mu} = \frac{\bar{r}}{w_l \bar{l}} (1 - \alpha) \quad (6)$$

$$\begin{aligned} \frac{\partial \ln(\bar{\mu})}{\partial \ln(L)} &= \frac{\partial \ln(\bar{r}/(w_l \bar{l}))}{\partial \ln(L)} \\ \frac{\partial \ln(\bar{\mu})}{\partial \ln(N)} &= \frac{\frac{\partial \ln(\bar{r}/(w_l \bar{l}))}{\partial \ln(L)}}{\frac{\partial \ln(N)}{\partial \ln(L)}}, \end{aligned} \quad (7)$$

where the last step applies the chain rule. The elasticity $\frac{\partial \ln(\bar{\mu})}{\partial \ln(N)}$ measures the decline in weighted-average markup as a result of an increase in competition (number of firms).

Since markups are not directly observed, the theoretical model suggests that the LHS can be estimated by the inverse wage-share elasticity $\frac{\partial \ln(\bar{r}/(w_l \bar{l}))}{\partial \ln(L)}$ and number of firm elasticity $\frac{\partial \ln(N)}{\partial \ln(L)}$; equation (7) has an IV interpretation. In the model, change in market size is the source of exogenous variation, and the number of firms N is the endogenous variable. So, the elasticity of markup to number of firms can be estimated by the ratio of the “reduced form”² over the first stage. The estimate of the first stage is the elasticity of number of firms to market size, and the estimate of the “reduced form” is the elasticity of the inverse wage share to market size. Note that in the case of homogeneous firms as in Campbell and Hopenhayn (2005), the same metric can be defined simply from the price-to-marginal cost expression:

$$\mu = \frac{p}{mc} = \frac{r}{mc \, q} = \frac{r}{\frac{1}{c} \left(\frac{w_k}{\alpha}\right)^\alpha \left(\frac{w_l}{1-\alpha}\right)^{1-\alpha} L \, Q \, D(.)} = \frac{r}{w_l \, l} (1 - \alpha), \quad (8)$$

As in De Loecker, Eeckhout, and Unger (2020), the same intuition holds; markups are the gaps between the output elasticities of input and the input cost shares.

There is a close connection between my measure and the recent literature on markups estimation. Equation (6) is very similar to the markups estimation equation in De Loecker, Eeckhout, and Unger (2020). However, by taking log and derivative of the average markup, the output elasticity of input drops out. Therefore, one doesn’t need to estimate the production functions with my method. This is an advantage because the production approach relies on revenue data instead of quantity data, which leads to certain methodological issues as mentioned in the introduction. In addition, the production approach also needs input data on at least one flexible input; what kind of input is appropriate has been subject to debates and the type chosen also imposes a stricter requirement on data availability. My approach doesn’t require a particular type of input; therefore, it can be applied to wage data, which is usually available in many datasets, such as the publicly available Economics Censuses. An implicit assumption that makes the input elasticity of output disappear when taking the partial derivative is that all firms share the same production function technologies; while this is unlikely to be true in reality, it is a typical assumption when

²Technically, regressing the inverse-wage share on population is not a reduced form since the outcome variable of interest from equation (7) is weighted-average markup.

estimating the production functions.

Discussion of other often-used measures: While concentration is straightforward to measure, it is not informative about toughness of competition (see e.g., Syverson (2019)). Specifically, using concentration as a measure of market power relies implicitly on the mechanism of the standard Cournot model where there is a positive relationship between concentration and average industry profitability. Outside of this context, concentration could be a noisy indicator. For example, I parameterize my model by imposing a Kimball (1995) demand system and the Klenow and Willis (2016) specification to generate an increase in average demand elasticity by increasing σ (a parameter from the utility function that governs the elasticity of demand function in this example); see Appendix C for details of this specification. Figure 1 shows that as elasticity of demand from the residual demand function that each firm faces increases, market concentration (measured by the Herfindahl-Hirschman Index) increases consequently.

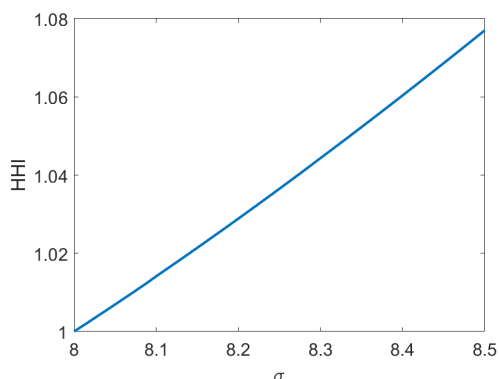


Figure 1: Comparative statics: Changes in concentration to changes in elasticity of demand

On the other hand, the textbook definition of market power is the ability of a firm to set its optimal price above marginal cost; hence markup or the Lerner index has been the traditional measure of market power. However, equilibrium markups could be high because competition is tough since entrants know that if new firms enter the market, incumbents will reduce markups significantly to compete. In this sense, the decline in markup when new competitors enter the market is a better measure of competition.

3. Data

The main datasets for this study are the Standard Statistical Establishment List (SSEL) and the Business Register (BR). These datasets report revenue, employment, and payroll information from all firms in the U.S. Essentially, both of these programs source data from the Internal Revenue Service; the SSEL covered data until 2001 and then was replaced by the BR. For a more detailed description of these datasets, see Haltiwanger, Jarmin, Kulick, Miranda, Penciakova, and Tello-Trillo (2019). Since revenue data in the SSEL is reported at the firm-level due to the nature of tax reporting, I use the Longitudinal Business Database (LBD) to compute the establishment-level payroll shares to re-distribute revenue to the plant level. Average revenue and payroll in a market are then simply the averages of plants' revenue and payroll in a specific county - 6 digit NAICS industry, and the number of plants is defined analogously. While I have access to the complete data from 1997 to 2016, in 2001 and 2002 there is a substantial portion of firms without reported revenue. Therefore, I exclude these two years from my study.

As mentioned earlier, I restrict my sample to non-tradable industries only. To identify these industries, I rely on the intuition of Jensen and Kletzer (2005) that tradable industries tend to display geographical concentrations in production activities. Specifically, I simply sort the industries by the number of counties that an industry has at least one operating plant in. I then pick the top 25 percentiles as non-tradable industries. The main results are not sensitive to this cut-off level. This selection criteria implies that the Restaurant industry is considered non-tradable because there are restaurants in almost all counties, while Data Processing, Hosting and Related Services are considered tradable since most firms locate in the Silicon Valley.

I construct the set of demographic control variables to include the share of African-Americans, share of working-age population (age 25-64), residential rent, and average personal income from various sources. I obtained demographics data mainly from the Decennial Census program, and home price data as a proxy for rent from Zillow. Population and income data come from the Bureau of Economic Analysis. Since these data are already reported at the county level, it is straightforward to merge them with the Census firm data.

In addition to using county, I also use MSA as a geographical definition of market. Since I have access to the micro-level data on the establishment side, I simply aggregate

revenue, payroll and number of establishments to the MSA level instead of the county level. In addition, since all the control variables are only available at the county level, I aggregate these variables to the MSA level by taking weighted-averages using county population share as weights.

In the robustness exercises, I use the County Business Patterns (CBP) data because it has fewer selection issues than the public version of the Economic Census. To try out a different measure of market size, I use the Input-Output (IO) table from the BEA as the last robustness check. I describe how these datasets are used at the corresponding sections of the paper.

4. Empirical estimation

To estimate $\frac{\partial \ln(\bar{\mu})}{\partial \ln(N)}$ (markup elasticity), I estimate the elasticity of inverse-wage share and the elasticity of number of establishments to market size (a pair of six digit NAICS - county) using the following equation:

$$y_{ilt} = \alpha + \beta m_{lt} + (m_{lt} * I_i * T_t)\gamma_1 + x_{ilt}\gamma_2 + \psi_i + \phi_t + \epsilon_{ilt}, \quad (9)$$

where y_{ilt} denotes the log outcome variable of interest, which are the average inverse wage share and number of establishments for industry i , at location (county in the baseline estimation) l in year t . m_{lt} is the log population, a measure of market size. I_i and T_t are the industry and year dummies, and ψ_i and ϕ_t are the industry and year fixed effects, respectively. x_{ilt} is a vector of controls including log average home value, log average income per capita, share of population between 25 and 54 years old, share of African-American population and share of people with at least a bachelor degree. The log average home value is a proxy for fixed costs, which are certainly important for the average plant size and number of establishments. For example, if commercial rent is high, it may imply there are fewer plants and they are bigger on average. Other demographics controls account for variation in business activities that are not due to variation in market size, such as the availability and skill level of the labor force and the budget size of consumers.

I first estimate (9) without the industry dummies in the interaction term and the industry FEs. The coefficient γ_1 is then the weighted-average across industries of cross-sectional effects of market size relative to the base year. This is the baseline specification

in my study. Then, to account for heterogeneity across industries, I allow the slopes of market size and intercepts to vary across industries by including the industry dummies both outside and inside the interaction term.

Simply estimating (9) using OLS would lead to biased estimates due to endogeneity concerns. First of all, omitted variable bias results from confounding factors that could not be controlled for, such as tax incentives. Firms may place their units of production in locations with favorable tax/subsidy policies that local governments create. Second, population data is not perfectly measured every year. Actual counts of population are only conducted in decennial years. Annual population is then updated accordingly using birth/death and migration data. To the extent that this leads to measurement error in the population variable, it could lead to attenuation bias. Finally, one would worry about reverse-causality. People migrate to new places for economic opportunities that are likely to correlate with business actives such as the number of firms in that specific location. In other word, there is a causal effect of x on y and also y on x . On the surface, because there are two different outcome variables of interest, there might be different identification threats. This amounts to coming up with different omitted variables that correlate with the wage share but not with the number of plants and vice versa. It is hard to come up with one such example.

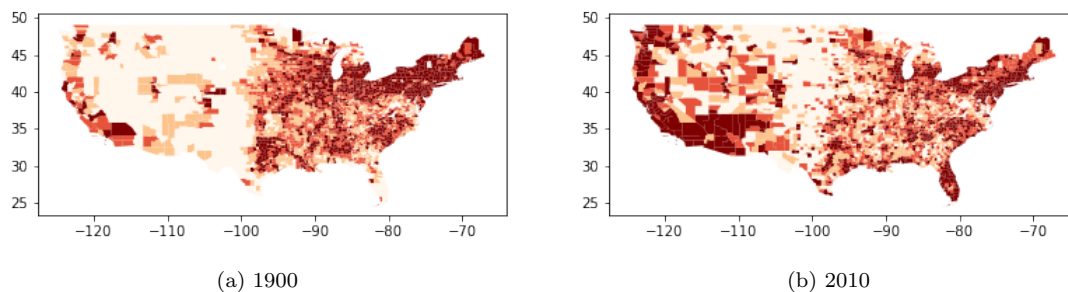


Figure 2: Historical vs current spatial distribution of US population

Identification strategy: To account for these endogeneity issues, I use an instrumental variable approach. Specifically, I generate a predictor of county population at a given time t by interacting the spatial distribution of the county population in 1900 with

the U.S. population in year t . Formally,

$$\text{predicted_population}_{lt} = \text{population_share}_{l1900} * \text{US_population}_t \quad (10)$$

where $\text{population_share}_{l1900}$ is the share of population in year 1900 at county l , and US_population_t is simply the population of the U.S. in year t . I use 1900 population data with longitudinally consistent county boundaries from Eckert, Gvirtz, Liang, and Peters (2020). Figure 2 plots the spatial distribution of population in the U.S. in 1900 and 2010. The darker regions indicate more populated counties; this figure shows that there have been substantial changes in population distribution after a century. For example, the West Coast has become more concentrated and the New England area has become relatively less concentrated. On one hand, these differences suggest that factors that attracted people in 1900 did not persist. On the other hand, I later show that the instrument has strong predictive power, suggesting relevance. The exogeneity assumption of this instrument is that the spatial distribution of people in 1900 is independent of economic conditions in year t . This condition might be violated in certain places with great natural resource deposits or proximity to docks. For example, the first oil field was discovered in the Los Angeles area in 1903, and by 1930, the region was producing a quarter of world's total oil supply. During the same time period, the population of the area grew from around 50,000 to over a million. Today, the Oil and Gas industry is still a large share of the region's economy. Therefore, oil deposits might be the same unobserved characteristics that explain the Los Angeles county population both in 1900 and today. As a result, I exclude some very large counties (top 5 percentiles) from my sample.

Since the interaction term in (9) leads to many equations for the first stages, I estimate the IV model using a control function approach following Wooldridge (2015). Specifically, in the first stage, I regress population on a full set of controls and the instruments. In the second stage, I regress the outcome variable on all the controls, the interaction term and the OLS residual obtained from the first stage. To account for the fact that the residuals are themselves estimates and the markup elasticity is a non-linear combination of regression coefficients, I bootstrap the system of equations to obtain the standard errors.

5. Results

Using the confidential Census data, I first present results for the baseline estimates at the county level and the industry heterogeneity exercise. In the next subsection, I present the same set of results at the MSA level. In the third subsection, I show that whether to weight the average markup or not does not change the main conclusion. Finally, I show a set of robustness checks that experiment with different definitions of market size, all of which generate outcomes that are consistent with the baseline findings.

5.1. *Main result*

In this section, I present the main empirical findings on how local competition has changed. Figure 3 shows the baseline result for non-tradable industries, in which all industries share a common slope within a year. In the left panel of figure 3, the dashed green line denotes the inverse wage share estimates, the dashed orange line denotes the number of establishment estimates and the solid blue line is the toughness of competition estimates from the OLS model. This figure shows that toughness of local competition decreased between 1997 to 2016, primarily driven by the inverse wage share elasticity and attenuated by the number of establishment elasticity. The red vertical intervals show the 95% confidence intervals, which indicate that the trend is statistically significant. The right panel of Figure 3 shows a similar trend from the IV model. Regarding the relevance of the instruments, the Cragg-Donald F-statistic is 333.5, suggesting the instruments are highly relevant.³

³Due to disclosure constraints, I use the public version of the Economic Census to report this statistic.

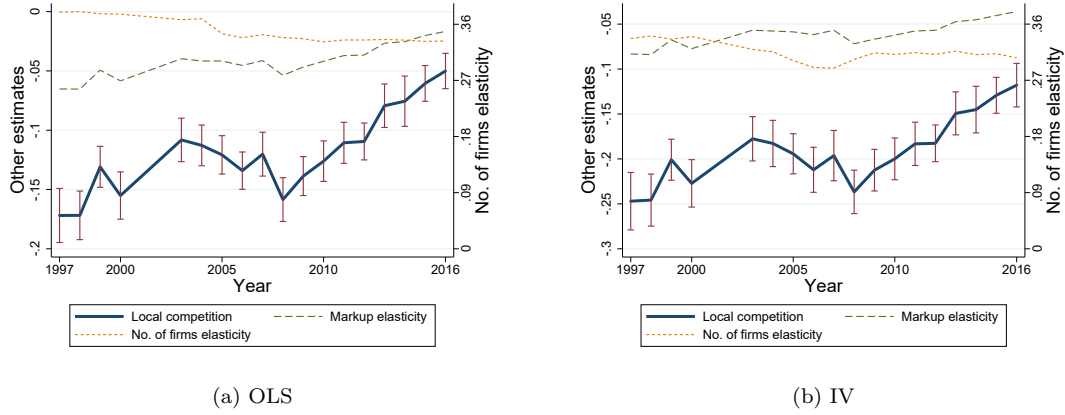


Figure 3: Baseline estimate (county-level)

While the baseline model provides a convenient summary or a weighted-average of the estimates across sectors, it is also important to look at the overall distribution to see if the previous results are driven by certain sectors. Figure 4 shows the mean and different percentiles of the estimates over time. The overall pattern over time seems to be consistent with those in figure 3, with the upward trend picking up only since 2008. While the trends are similar between the OLS specification (left panel) and the IV specification (right panel), the scales are very different. The 75th percentile is all positive in the left panel, which is inconsistent with the theoretical model. However, the IV specification shows that the whole 75th percentile is negative, which is consistent with my theory. These results suggest that markups have declined less in percentage terms over time when the number of competitors has increased.

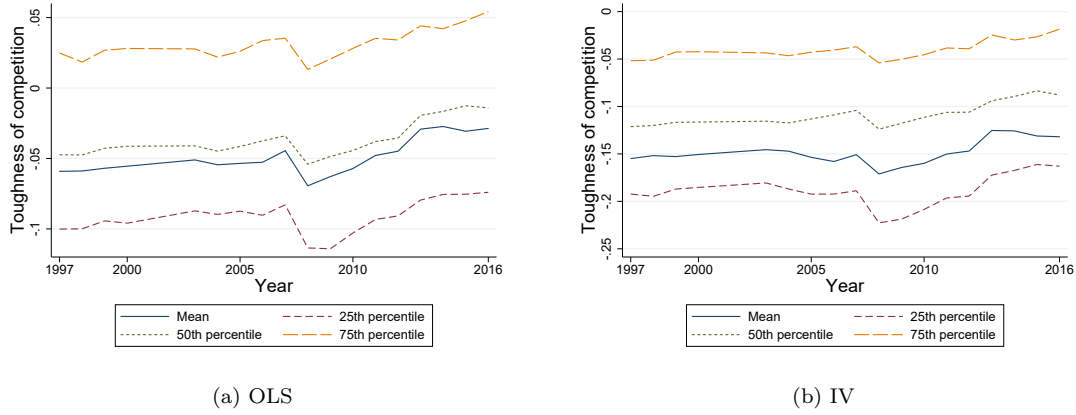
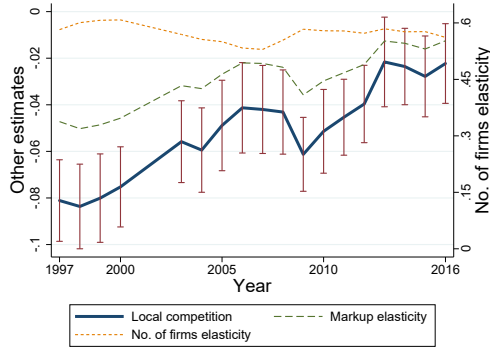


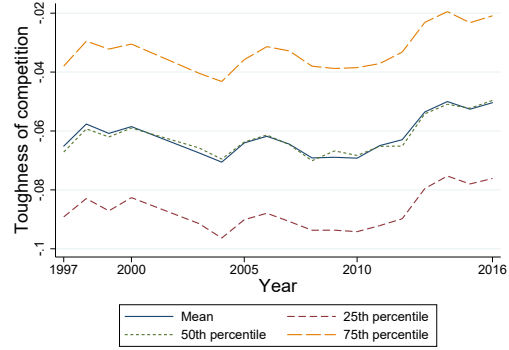
Figure 4: Distribution (across industries) of estimates

5.2. Using MSA instead of county

In the baseline results, I used county as a geographical definition of market. Since it is difficult to define a market, I try different geographical boundaries to see if the previous results are sensitive to this definition. Following Campbell and Hopenhayn (2005), I use MSA instead of county. Because some control variables are only available at the county level, I map them to the MSA level using a population-weighted average basis. The left panel of figure 5 shows the estimates from the IV model with common industry slope. Similar to the baseline results (county-level), the trend in toughness of competition is statistically significant, which is driven by the inverse wage share elasticity and slightly attenuated by the number of establishment elasticity. When looking at the distribution of estimates as in the right panel, the trend is less visible; however, the whole 75th percentile is below zero and the pattern in the left panel seems to be representative of the majority of industries.



(a) Weighted



(b) Un-weighted

Figure 5: MSA-level estimates

5.3. Weighted-average markup versus un-weighted average markup

All empirical results so far are based on equation (6), which equates the cost-weighted average markup to a function of the ratio of average revenue over average wage. This is convenient because one can estimate toughness of competition with just publicly available data. With access to the micro data, I can also estimate the decline in un-weighted average markup using the average of firm-level inverse wage share. Figure 6 below reports the results at the MSA level using the IV specification. Similar to findings in the previous sections, toughness of competition has declined over time (left panel), which is once again driven by the inverse wage share elasticity and attenuated by the number of establishment elasticity. The right panel shows a much more visible trend that seems to be shared by all industries.

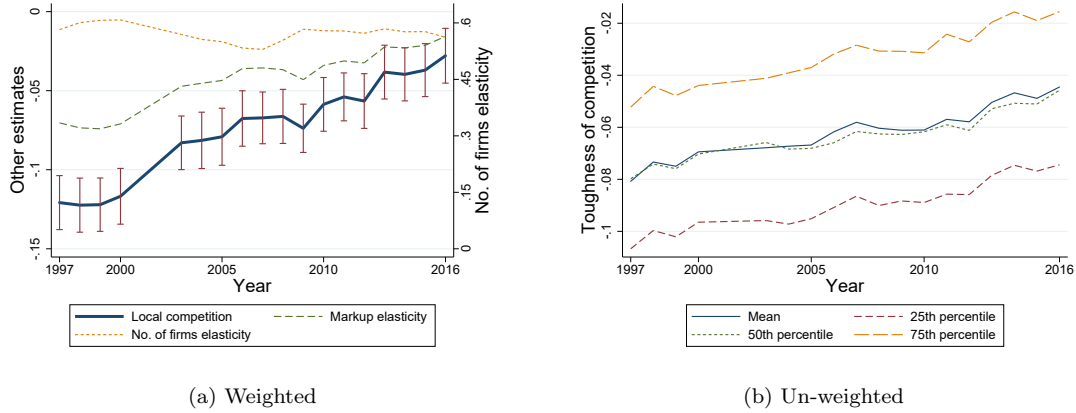


Figure 6: MSA-level, un-weighted estimates

5.4. Robustness checks

It is a strong assumption to define market size as the population for all goods and services. Therefore, in this subsection, I attempt to use more precise definitions of market size at the cost of generality by looking at some specific industries. I first follow Syverson (2004) in the ready-mixed concrete industry. Then, I use the Input-Output table to derive a measure of market size.

5.4.1. Robustness test: Using Syverson (2004)'s measure of market size

Many papers, such as Asplund and Nocke (2006), and Campbell and Hopenhayn (2005), have used population as a measure of market size. Using population gives an advantage because it is available for many geographical locations and years, especially in this study which aims to cast a broader net instead of focusing on a particular industry. However, the general population are not direct users of certain goods; therefore, in this section, I perform a robustness exercise by measuring market size following Syverson (2004). Specifically, for the ready-mixed concrete industry (2012 NAICS code = 327320), market size is defined as the number of construction workers (2012 NAICS code = 23) per square mile in a county.

Since reporting results for a single industry is likely to violate the disclosure requirement of the Census, in this section, I use the County Business Patterns (CBP) data as an

alternative.⁴ The CBP doesn't include revenue information; therefore, I only report results for the number of establishment elasticity in this section. Figure 7 plots the estimate of log number of establishments on log population (blue line) versus number of construction workers per square mile (red dashed line) using an OLS specification. Although the magnitude of estimates is different, the trends are remarkably similar. This result suggests that population is a reasonable measure of market size even in this specific context.

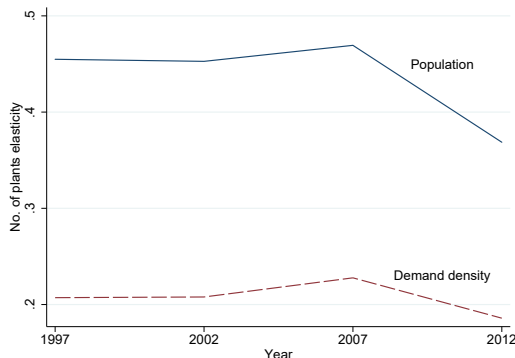


Figure 7: Elasticity of firm size to population and demand density

5.4.2. Robustness test: Measure market size using the Input-Output table

As mentioned earlier, using population as market size might be inaccurate in certain cases where the direct end users are not the general population. In the concrete example above, population seems to do a decent job as a proxy. However, to account for this in a boarder way, in this section I use the IO table from the BEA to construct a market size measure for each industry-location pair. Specifically, from the IO table, for each industry i that sells to the set of industry J , the market size for industry i in a given location is the sum of the number of workers in J . I use the number of workers instead of revenue simply due to endogeneity concerns; e.g., higher revenue might not reflect bigger market size but higher average cost or markups.

Figure 8 plots the estimates for each industry-year (census years from 1997-2017) using the baseline specification for each measure of market size. Most of the estimates are

⁴I did not use the public version of Economic Census data since it has many truncation issues due to confidentiality concerns.

on the 45-degree line, which suggests that population is just as good a measure of market size as the IO-based variable.

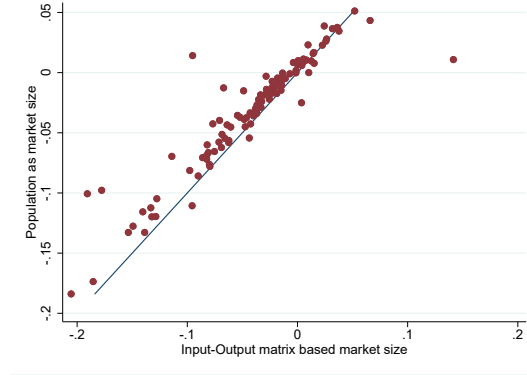


Figure 8: Elasticity of firm size to population and demand density

6. Conclusion

This study is the first to estimate toughness of competition over time. I motivate this measure using a very general class of model, then show that my measure is very closely related to recent studies in the markup estimation literature; however, my approach is not subject to the typical production functions estimation obstacle. Instead, I need minimal data to estimate toughness of competition. I find that toughness of local competition decreased between 1997 and 2016. This finding is robust to a number of checks. In future work, I first hope to understand what could explain this change and extend my work to tradable goods as well.

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Appendix A. Derivation

This section shows the derivation of the toughness of competition (7) in detailed steps. Denoting c^* as the corresponding cutoff efficiency level, ex-post average revenue is given by:

$$\bar{r} = \int_0^{c^*} \mu \left(\frac{P}{mc} \right) (mc) L Q D \left(\mu \left(\frac{P}{mc} \right) / \frac{P}{mc} \right) g^*(c) dc$$

Using the change of variable: $c = \frac{P}{\psi\nu}$ and $dc = -\frac{P}{\psi\nu^2} d\nu$ where $\psi = \left(\frac{w_k}{\alpha} \right)^\alpha \left(\frac{w_l}{1-\alpha} \right)^{(1-\alpha)}$

$$\begin{aligned} \bar{r} &= -\psi L Q \int_\infty^1 \mu(\nu) \frac{P}{\psi\nu} D(\mu(\nu)/\nu) g^* \left(\frac{P}{\psi\nu} \right) \frac{P}{\psi\nu^2} d\nu \\ \bar{r} &= L Q \frac{P^2}{\psi} \int_1^\infty \mu(\nu) D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{\psi\nu} \right) d\nu \end{aligned} \quad (\text{A.1})$$

Recalling \bar{TC} is the average cost used for production,

$$\begin{aligned} \bar{TC} &= \int_0^{c^*} \psi q c g(c) dc \\ &= \psi \int_\infty^1 L Q D(\mu(\nu)/\nu) \frac{P}{\psi\nu} g^* \left(\frac{P}{\psi\nu} \right) \frac{P}{\psi\nu^2} d\nu \\ \bar{TC} &= L Q \frac{P^2}{\psi} \int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{\psi\nu} \right) d\nu \end{aligned}$$

Therefore, the weighted-average markup is:

$$\begin{aligned} \bar{\mu} &= \int_0^{c^*} \frac{TC}{\bar{TC}} \mu \left(\frac{P}{mc} \right) g^*(c) dc \\ \bar{\mu} &= \int_0^{c^*} \frac{\psi q c}{L Q \frac{P^2}{\psi} \int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{\psi\nu} \right) d\nu} \mu \left(\frac{P}{\psi c} \right) g^*(c) dc \\ \bar{\mu} &= \frac{\int_1^\infty D(\mu(\nu)/\nu) \mu(\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{w\nu} \right) d\nu}{\int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{w\nu} \right) d\nu} \end{aligned} \quad (\text{A.2})$$

Since

$$\bar{l} = \int_0^{c^*} \left(\frac{1-\alpha}{\alpha} \frac{w_k}{w_l} \right)^\alpha q c g(c) dc$$

$$= \left(\frac{1-\alpha}{\alpha} \frac{w_k}{w_l} \right)^\alpha L Q \frac{P^2}{\psi^2} \int_1^\infty D(\mu(\nu)/\nu) \frac{1}{\nu^3} g^* \left(\frac{P}{w\nu} \right) d\nu \quad (\text{A.3})$$

Combining equations (A.1), (A.2) and (A.3) gives:

$$\bar{\mu} = \frac{\bar{r}}{w_l \bar{l}} (1 - \alpha) \quad (\text{A.4})$$

$$\frac{\partial \ln(\bar{\mu})}{\partial \ln(L)} = \frac{\partial \ln(\bar{r}/(w_l \bar{l}))}{\partial \ln(L)}$$

Applying Chain-rule gives,

$$\frac{\partial \ln(\bar{\mu})}{\partial \ln(N)} = \frac{\frac{\partial \ln(\bar{r}/(w_l \bar{l}))}{\partial \ln(L)}}{\frac{\partial \ln(N)}{\partial \ln(L)}}, \quad (\text{A.5})$$

Appendix B. A version with trade cost

In the main text, both the theoretical model and empirical studies focus only on non-tradable goods. In this section, I show that this study could be extended to tradable goods as well by deriving the toughness of competition measure from an economy where goods can be sold non-locally subject to some arbitrary trade cost τ . For simplicity and without loss of generality, I assume that labor is the only factor of production and productivity is Pareto distributed everywhere.

There are two regions i and j that are different in market size and wage rate. The average revenue of firms generated from selling in j is given by:

$$\begin{aligned} \bar{r}_j &= \frac{N_{ij} \int_0^{c_{ij}^*} r_{ij} k c^{k-1} (c_{ij}^*)^{-k} dc + N_{jj} \int_0^{c_j^*} r_{jj} k c^{k-1} (c_j^*)^{-k} dc}{N_j} \\ \int_0^{c_{ij}^*} r_{ij} k c^{k-1} (c_{ij}^*)^{-k} dc &= \\ \int_0^{c_{ij}^*} \mu_{ij} \left(\frac{P_j}{w_i \tau_{ij} c} \right) (w_i c) L_j Q_j D \left(\mu_{ij} \left(\frac{P_j}{w_i \tau_{ij} c} \right) / \frac{P_j}{w_i \tau_{ij} c} \right) k c^{k-1} (c_{ij}^*)^{-k} dc & \quad (\text{B.1}) \\ &= w_i L_j Q_j k (c_{ij}^*)^{-k} \int_1^\infty \mu_{ij}(\nu) \left(\frac{P_j}{w_i \tau_{ij} \nu} \right)^k D \left(\mu_{ij}(\nu)/\nu \right) \left(\frac{P_j}{w_i \tau_{ij} \nu^2} \right) d\nu \end{aligned}$$

Since $c_{ij}^* = \frac{c_j^* w_j}{w_i \tau_{ij}}$ (the difference in cutoff productivity of local firms and outside firms only result from transportation cost and difference in wage rate),

$$\begin{aligned} &= w_i L_j Q_j k \left(\frac{c_j^* w_j}{w_i \tau_{ij}} \right)^{-k} \left(\frac{c_j^* w_j}{w_i \tau_{ij}} \right)^{k+1} \int_1^\infty \mu_{ij}(\nu) D\left(\mu_{ij}(\nu)/\nu\right) \nu^{-2-k} d\nu \\ &= w_i L_j Q_j k c_j^* \left(\frac{w_j}{w_i \tau_{ij}} \right) \int_1^\infty \mu_{ij}(\nu) D\left(\mu_{ij}(\nu)/\nu\right) \nu^{-2-k} d\nu \end{aligned}$$

Similarly,

$$\int_0^{c_j^*} r_{jj} k c^{k-1} (c_j^*)^{-k} dc = w_j L_j Q_j k c_j^* \int_1^\infty \mu_{jj}(\nu) D\left(\mu_{jj}(\nu)/\nu\right) \nu^{-2-k} d\nu$$

Therefore,

$$\bar{r}_j = \frac{w_j L_j Q_j k c_j^* \phi\left(N_{ij}\left(\frac{w_j}{w_i \tau_{ij}}\right) + N_{jj}\right)}{N_j} \quad (\text{B.2})$$

where ϕ denotes the integral term. Cost-weighted average markup of firms selling in j is given by:

$$\bar{r}_j = \frac{N_{ij} \int_0^{c_{ij}^*} \frac{l(c)_{ij}}{\bar{l}_j} \mu_{ij} k c^{k-1} (c_{ij}^*)^{-k} dc + N_{jj} \int_0^{c_j^*} \frac{l(c)_{jj}}{\bar{l}_j} \mu_{jj} k c^{k-1} (c_j^*)^{-k} dc}{N_j}$$

where \bar{l}_j is the average number of workers employed to produce goods selling in j .

$$\begin{aligned} \int_0^{c_{ij}^*} \frac{l(c)}{\bar{l}_j} \mu_{ij} k c^{k-1} (c_{ij}^*)^{-k} dc &= \int_0^{c_{ij}^*} \frac{c L_j Q_j D\left(\mu\left(\frac{P_j}{w_i \tau_{ij} c}\right) / \frac{P_j}{w_i \tau_{ij} c}\right)}{\bar{l}_j} \mu\left(\frac{P_j}{w_i \tau_{ij} c}\right) k c^{k-1} (c_{ij}^*)^{-k} dc \quad (\text{B.3}) \\ &= k (c_{ij}^*)^{-k} \frac{L_j Q_j}{\bar{l}_j} \int_1^\infty D\left(\mu(\nu)/\nu\right) \mu(\nu) \left(\frac{P_j}{w_i \tau_{ij} \nu}\right)^k \left(\frac{P_j}{w_i \tau_{ij} \nu^2}\right) d\nu \\ &= k (c_{ij}^*)^{-k} \frac{L_j Q_j}{\bar{l}_j} \left(\frac{P_j}{w_i \tau_{ij}}\right)^{k+1} \int_1^\infty D\left(\mu(\nu)/\nu\right) \mu(\nu) \nu^{-2-k} d\nu \end{aligned}$$

$$\begin{aligned}
&= k \left(\frac{c_j^* w_j}{w_i \tau_{ij}} \right)^{-k} \frac{L_j Q_j}{\bar{l}_j} \left(\frac{c_j^* w_j}{w_i \tau_{ij}} \right)^{k+1} \int_1^\infty D\left(\mu(\nu)/\nu\right) \mu(\nu) \nu^{-2-k} d\nu \\
&= k c_j^* \left(\frac{w_j}{w_i \tau_{ij}} \right) \frac{L_j Q_j}{\bar{l}_j} \int_1^\infty D\left(\mu(\nu)/\nu\right) \mu(\nu) \nu^{-2-k} d\nu
\end{aligned}$$

Similarly,

$$\int_0^{c_j^*} \frac{l(c)}{\bar{l}_j} \mu_{jj} k c^{k-1} (c_j^*)^{-k} dc = k c_j^* \frac{L_j Q_j}{\bar{l}_j} \int_1^\infty D\left(\mu(\nu)/\nu\right) \mu(\nu) \nu^{-2-k} d\nu$$

Therefore,

$$\bar{\mu}_j = \frac{\frac{L_j Q_j}{\bar{l}_j} k c_j^* \phi\left(N_{ij}\left(\frac{w_j}{w_i \tau_{ij}}\right) + N_{jj}\right)}{N_j} \quad (\text{B.4})$$

Combining (B.2) and (B.4),

$$\bar{\mu}_j = \frac{\bar{r}_j}{w_j \bar{l}_j} \quad (\text{B.5})$$

Therefore,

$$\frac{\partial \ln(\bar{\mu}_j)}{\partial \ln(N_{ij} + N_{jj})} = \frac{\frac{\partial \ln(\bar{r}_j / w_j \bar{l}_j)}{\partial \ln(L_j)}}{\frac{\partial \ln(N_{ij} + N_{jj})}{\partial \ln(L_j)}} \quad (\text{B.6})$$

Equation B.6 looks very similar to its counterpart (7) in the main text; however, estimating B.6 requires very detailed data. For example N_j denotes the number of firms located in j that sells locally plus the number of firms located in i that sells in j . In the limit, when $\tau \rightarrow \infty$, equation (B.6) collapses to equation (7). Therefore, while it is theoretically possible to estimate toughness of competition for tradable goods, it is not practically possible, at least without imposing further assumptions. I leave this task as a topic for future research.

Appendix C. Kimball demand specification simulations

The data generating process is given by a monopolistic competition model with Kimball demand as in Amiti, Itskhoki, and Konings (2019). The economy consists of many isolated regions of different sizes as a result of infinite trade cost. In each region, firms are

different in their productivity and charge variable markups based on their market shares. Aggregate consumption Q is given by:

$$\begin{aligned} \int_{\omega \in \Omega^*} \Upsilon\left(\frac{q_\omega}{Q}\right) d\omega &= 1 \\ \text{s.t.} \\ \int_{\omega \in \Omega^*} p_\omega q_\omega &= wL \end{aligned} \tag{C.1}$$

where Ω^* is the set of available varieties to be determined in equilibrium. $\Upsilon(1) = 1$, $\Upsilon(q) > 0$, $\Upsilon(q)' > 0$ and $\Upsilon(q)'' < 0$. The resulting demand function for each varieties is:

$$q_\omega = \psi\left(\frac{p_\omega}{P/D}\right) Q$$

where $\psi(.) = (\Upsilon(.))'^{-1}$ and D is a demand index, P is the price index.

Firms produce unique variety with labor as the only input under constant return to scale technology. The problem of the firm is given by:

$$\max_{p(a)} \pi(a) = p(a) q(a) - w l(a)$$

s.t to the demand equation C.1 and the production function where $a \sim G(a)$. Denoting $\epsilon_d(.)$ as the elasticity of demand, the resulting first order condition is

$$p(a) = \frac{\epsilon_d(.)}{\epsilon_d(.) - 1} \frac{w}{a}. \tag{C.2}$$

The mass of firms N in each regions is pinned down by the free entry condition. In each region, all entrants pay sunk entry costs f_e , then draw their productivity. Firms that draw below a^* immediately shut down. Formally,

$$\int_{a^*}^{\infty} \pi(a) dG(a) = f_e \tag{C.3}$$

To solve the model, I adopt the Klenow and Willis (2016) specification for $\Upsilon(.)$, where

$$\Upsilon(q) = 1 + (\sigma - 1) \exp\left(\frac{1}{\epsilon}\right) \epsilon^{\frac{\sigma}{\epsilon} - 1} \left[\Gamma\left(\frac{\sigma}{\epsilon}, \frac{1}{\epsilon}\right) - \Gamma\left(\frac{\sigma}{\epsilon}, \frac{q^{\frac{\epsilon}{\sigma}}}{\epsilon}\right) \right], \tag{C.4}$$

for $\sigma > 1$, $\epsilon \geq 0$ and

$$\Gamma(s, x) = \int_x^{\infty} t^{s-1} e^{-t} dt$$

denotes the upper incomplete Gamma function. With this specification,

$$\psi(x) = \left(1 - \epsilon \ln\left(\frac{\sigma}{\sigma-1}x\right)\right)^{\frac{\sigma}{\epsilon}}, \quad (\text{C.5})$$

and the elasticity of demand is

$$\epsilon_d(x) = \frac{\sigma}{1 - \epsilon \ln\left(\frac{\sigma}{\sigma-1}x\right)} \quad (\text{C.6})$$

The choke price is

$$p^* = \exp\left(\frac{1}{\epsilon}\right) \frac{\sigma-1}{\sigma} \frac{P}{D} \quad (\text{C.7})$$

which pins down the zero cutoff profit condition. The equilibrium is the set of prices $\{p_\omega\}$, P and quantity $\{q_\omega, Q\}$ such that the output market and labor market clear, the free-entry condition holds.

To solve for the general equilibrium, I guess $\frac{P}{D}$ to solve for the optimal pricing functions with a root finding algorithm, which then gives the price index P and subsequently Q and q_ω from the budget constraint and demand functions. The number of firms is set such that the labor market clears, I then check the market clearing, free-entry condition and update $\frac{P}{D}$ accordingly. I choose parameters of the model following commonly used values in the literature. I set $\sigma = 9$, $\epsilon = 2.6$, $f_e = 0.01$. I let productivity to be drawn from a *Pareto*(3.5, 0.8) and *Log Normal*(2, 0.9) alternatively.